Study of the Π Network as the Compound Slot Equivalent Circuit Model

Ignacio Montesinos-Ortego[*+], Miao Zhang[+], Manuel Sierra[*], Jiro Hirokawa[+], Makoto Ando[+]

Radiation Group – Dept. of Signals, Systems and Radiocommunications Technical University of Madrid (UPM),
Complutense Avenue, 28040 Madrid, Spain
nacho@gr.ssr.upm.es

[+] *Dept. of Electrical and Electronic Eng., Tokyo Institute of Technology*
Meguro-ku, Tokyo.152-8552, Japan
miao@antenna.ee.titech.ac.jp

Abstract—. **A combination of Method of Moments (MoM) and compound slot Equivalent Circuit Model for linear array design is presented in this document. From the S Matrix of the single element, the more suitable network for its characterization is analyzed and selected. Then according to the radiation requirements of the desired array, the elements are designed and then properly connected by means of Forward Matching Procedure (FMP), which takes into account impedance matters in order to keep the input matched at the designing frequency. Comparison between HFSS simulations and MoM-FMP results are also presented. First part of this work was introduced in [1][2] but a summary is included here to make the understanding easier.**

I. Introduction

Recently, well known applications like military and defence, biomedical, weapons detection, weather prediction are demanding more and more reliable communications systems and techniques in the millimetric and sub-millimetric frequency bands. Related to this, slot antennas are frequently used as the main radiator due to their low losses and reliability at high frequencies. In one hand, longitudinal and transverse slots in the wide face of a rectangular waveguide are easy to find as radiators in main planes and their equivalent circuit models are known [3] and they have been successfully used for array design [4][5][6]. In the other hand, compound slot (inclined and displaced from the axis of the waveguide) can be found as coupling elements in multi-layered antennas and occasionally as radiators [7]. Although the use of equivalent circuit for this last kind of slot is not recommended for the array design in [8], a new procedure and its results will be presented here.

The final purpose of this work is to design a longitudinal polarized bi-dimensional array using the equivalent circuit model. In order to achieve a low side-lobe-level, it is intended to radiate in the diagonal planes, $\varphi = 45^{\circ}$ or $\varphi = 135^{\circ}$. So that, forty-five degrees compound slots, separated approximately $\lambda_g/2$ from each other, will be employed for a longitudinal sub-array design. Then, multiple sub-arrays will be connected by means of a multilayered feeding network. Once the 2D grouping is formed, it will be rotated and the rhombus-shaped physical distribution will enhance the radiation properties. In addition to this, a specific non-uniform taper can be employed to reduce the radiation in undesired angles.

MoM-FMP technique can be divided in two different parts. Using MoM, the single element is analysed for different parameter values, generating a database that will be employed by FMP to synthesize an input-matched array with a requested aperture distribution.

II. Compound Slot Structure

In Fig. 2 the main parameters of the compound slot are shown. Offset (D) is the distance between centre of the slot and the centre of the waveguide; l is the length and width of the slot; the tilting angle (θ) is defined as the angle between the longitudinal axis of the waveguide and the longitudinal axis of the slot. Besides, the waveguide width and height, w and h respectively, will determine the propagation constant γ of the feeding mode and its cutoff frequency.

III. Compound Slot Analysis

In this paragraph the analysis of the single element is presented. First, according to the working frequency of the project, radiation performance and dimensions of the array, waveguide height and width as well as tilting angle are fixed ($w = 0.65\lambda_0$, $h = 0.325\lambda_0$, $\theta = 45^{\circ}$). In addition to this, according to the limitations in the manufacturing stage, the thickness of the waveguide wall that sets the slot depth t is also pre-established ($t = 0.06\lambda_0$). The waveguide width will be responsible of the propagation constant of the fundamental mode and for the attenuation constant of the first order mode. From another point of view, it is important to take into account that the width of the waveguide will affect the transversal separation of elements and consequently, the grating lobe level of the bi-dimensional array. Because of the physical disposition and shape of the slot, this limitation in the waveguide dimension causes that sometimes really small offsets are needed

Second, a two-dimensional parameter sweep (D, l) in a band of frequencies centred in 61.5GHz is carried out and [S], [Z] and [Y] matrices are obtained using Method of Moments (MoM) [4]. From them, components values are calculated at the centre frequency. The different behaviour of the preselected structures regarding offset and length parameter value will determine which one is the more suitable for the slot characterization in this project.

IV. CIRCUIT CHARACTERIZATION

The offset analysis of the S-parameters of the single element reveals that the slot can be regarded as a reciprocal non-symmetrical two-ports device. The equivalent networks must share the same property, so that T and Π network were deeply studied.

A. T - Network

T- network is composed of shunt admittance in between two serial impedances. The offset analysis shows that for large values, real part of serial impedances is zero and imaginary parts are conjugated, so that shunt admittance is the responsible of the radiation. At the resonance condition, susceptance becomes zero and slot does not alter the feeding wave phase.

The Z matrix is used to determine the components value and as it can be seen in eq. (1), all its elements are dependent on the inverse of the shunt admittance. When the offset is decreased and the slot is closer to the centre of the waveguide, less power is radiated. According to the model, this means that the shunt element value is decreasing and reaches the zero value when the offset is null. Because of this, the elements of the Z matrix rapidly increase and go to infinity when D = 0. Following the circuit theory, if the shunt element is zero, it can be neglected and longitudinal elements can be combined. Doing this, the centre-inclined slot equivalent circuit presented in [3] is obtained.

Despite this issue, T network can be used as equivalent circuit as long as the offset is large enough without exceeding waveguide edges, as can be seen in [5] where a successful linear array of three elements in the X band is designed. In 2D arrays, the presence of grating lobes limits the width of the waveguide, so that small offsets are frequently used and T network is not recommended.

B. Π - Network

The Π-network contains serial impedance and two conjugated shunt admittances. Tilting angle determines if the shunt elements are purely imaginary or not. For wide angles, admittances become susceptances and all the radiated power can be ascribed to the serial element. On the contrary, as the tilting angle becomes smaller, the value of the elements of the Y matrix rises and, if the tilting angle is equal to zero, serial element becomes null and shunt elements can be combined, obtaining the well-known longitudinal slot equivalent circuit model [3]. For wide tilting angles and resonance condition, shunt elements are purely imaginary and series component is purely real.

According to the specifications of the project, tilting angle is fixed and equal to 45º, which provokes that shunt elements could be seen as susceptances. It is important to highlight that for this network mathematical singularities are not found for every value of the swept parameters. So that, Π- Network was chosen as the more suitable equivalent circuit model for this kind of slot.

$$[Y]=\begin{bmatrix} \frac{1}{Z^C}+Y^A & \frac{-1}{Z^C} \\ \frac{-1}{Z^C} & \frac{1}{Z^C}+Y^B \end{bmatrix};\quad [Z]=\begin{bmatrix} Z^A+\frac{1}{Y^C} & \frac{1}{Y^C} \\ \frac{1}{Y^C} & Z^B+\frac{1}{Y^C} \end{bmatrix} \quad (1)$$

V. COMPOUND SLOT POWER HANDLING AND Π- NETWORK

As it was explained in III, dimensions of the waveguide inclination angle θ and slot width and depth are fixed. Slot length l and offset D are swept and S parameters are obtained for each case. From them, the admittance matrix and components value are calculated. This reveals that for every length and offset, the real part of the shunt admittances is always zero, so that serial impedance is the responsible of the radiation.

The slot characterization diverges in two different ways. From a microwave point of view, compound slot can be seen as a lossy two-port network, where power is reflected to the source (P_{REF}), power is transmitted to the load (P_L) and power consumed (radiated) by the network (P_{RAD}).

From the circuit analysis, expressions of voltage and current in every point of the network are derived and a relation between them and the power can be established:

$$P_L=|S_{21}|^2 \qquad P_{REF}=|S_{11}|^2 \qquad P_{RAD}=1-|S_{11}|^2-|S_{21}|^2$$

$$P_L=re\left(Z_L\cdot|I_2|^2\right) \qquad P_{REF}=\left|\frac{Z_{IN}-Z_G}{Z_{IN}+Z_G}\right|^2 \qquad P_{RAD}=re\left(|i_c|^2\cdot Z_C\right) \quad (2)$$

Power can be calculated by means of the absolute value of the scattering parameters but their phases remain unknown. Using the equivalent circuit, the phase of the radiated and transmitted power can be calculated as the phase of the current that flows along the series impedance and load impedance, respectively.

In order to undoubtedly accept this Π- Network equivalent circuit model as valid, an additional study was carried out. Different tilting angles were analysed, observing that if inclination angle is large enough, Y^A and Y^B are pure imaginary and therefore they do not contribute to the radiation, as it is plotted in Fig. 4, where θ = -45º. As the tilting angle becomes closer to zero, the value of the real part of the shunt element grows and consequently, these elements start to consume/radiate power and must be taken into account (3). In Fig. 5, for θ = -5º, shunt elements Y^A and Y^B become active in the radiation phenomena, while contribution from Z^C rapidly decreases. Only for θ = 0º, the impedance becomes zero, allowing the combination of Y^A and Y^B in only one shunt element, which means that even for this case, Π network fulfils the physical conditions and can be used for modelling compound, centred-tilted, transverse and longitudinal slots.

Table 1 compares the suitability of both networks for being used as electrical model of different kinds of slots.

TABLE I
NETWORKS' SUITABILITY

Slot Kind	T - Network	Π - Network
Longitudinal	Applicable	Applicable
Transverse	NA	Applicable
Centre-Inclined	NA	Applicable
Compound	Applicable	Applicable

$$P_{RAD} = re\left(\frac{|V_1 - V_2|^2}{Z_C}\right) + re\left(|V_1|^2 \cdot Y_a\right) + re\left(|V_2|^2 \cdot Y_b\right) \qquad (3)$$

$$\phi_i = \angle i_i^C$$

For any value of the tilting angle, the agreement between S parameters analysis and circuit theory in terms of power is verified. Although in this document only one slot length is presented, the agreement is kept for any value of this parameter. As a conclusion, the proposed equivalent circuit model perfectly fulfils the physical compound slot behaviour and its employment for the design of 1D arrays is proposed.

Inside the previously explained analysis, a specific case must be independently treated. At resonance, the phase of the feeding mode inside the waveguide is not shifted when it faces the slot, which means that in the S parameters study, the phase of the S_{21} is equal to zero.

For every offset, the value of the length that provokes the slot to be at resonance is calculated, and (D_r, l_r) are called resonant offset and resonant length, respectively. Under this status, the same angle analysis is carried out and an interesting conclusion was derived. When the offset is set as D_r, the maximum power is radiated when the length of the slot is the resonant length. Fixing the inclination angle, for different resonant offsets, the resonant length varies, as well as the amount of radiated power. It has been proven that the tilting angle will condition the difference between the maximum and minimum radiated power at resonance. As it can be seen in Fig. 8, for the desired angle whatever values would be chosen, the radiated power will be almost the same. As the angle becomes smaller, the difference between the maximum and minimum value grows considerably. If the slot is parallel to the waveguide axis, the difference between radiation limits reaches the maximum, since it is zero for null offset and almost one if the slot is placed close to the edge.

As a result, since the normalized radiated power of each slot for the imposed tilting angle cannot be controlled, it is impossible to design the compound slot array at the resonance status.

VI. CASCADING PROCEDURE

Fulfilling the project requirements of amplitude distribution and input matching, all the slots are designed in terms of length and offset, taking into account impedance matters when they are placed inside an array. From each of them, the ABCD matrix are calculated and connected accordingly. The process can be briefly described as follows.

-First: using the circuit model, the first element's load impedance is calculated in order to match the input of the array for all the swept values, since the value of the needed input impedance is known and equal to the characteristic impedance of the waveguide at the frequency of analysis. Among all the possibilities, those that radiate the requested power are selected as potential candidates.

-Second: the input impedance of the second slot must correspond to the load impedance of the previous one, so that the input is kept matched. Of course the length of the connecting waveguide must be taken into account. Cases in which the slot radiates the requested power with a radiation phase inside an interval centred in the previous slot's radiation phase will become candidates for the second slot. For every candidate of the first slot, a series of candidates for the second slot are proposed.

-Third: the process described in the previous paragraph is repeated till the last slot.

-Fourth: besides the conditions of radiation power and phase that all the rest of elements need to fulfil, the last one is requested to have specific load impedance. In the real model, the waveguide is shorted and the short length must be less than $0.25\lambda_g$ to avoid the blocking phenomenon, so that load impedance must be pure imaginary and positive. Once the last element is selected, the sub-array design is completed.

-Fifth: ABCD matrices of slots and waveguides are cascaded and frequency analysis is carried out.

VII. RESULTS

Following the MoM-FMP output parameters, the model is then simulated using HFSS 10 and a good agreement is achieved between them (Fig. 10). The computation time of the FMP running on a Mac Book Pro 2.8GHz is the 14 min. The difference between expected and simulated values is due to the external coupling between adjacent elements, which has not been taken into account in the equivalent circuit calculation during the MoM stage.

VIII. ACKNOWLEDGEMENTS

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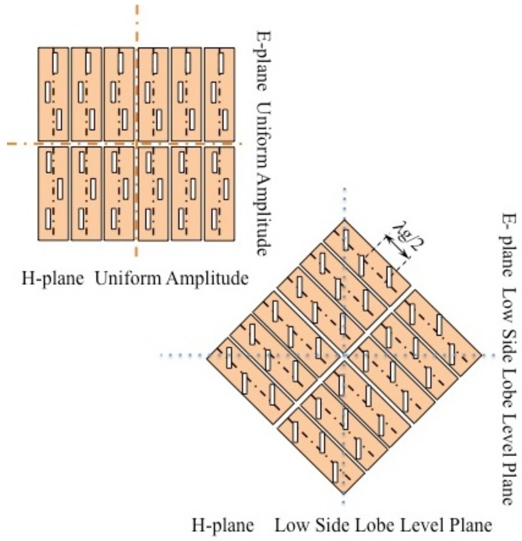

Fig. 1: 2D compound slot array

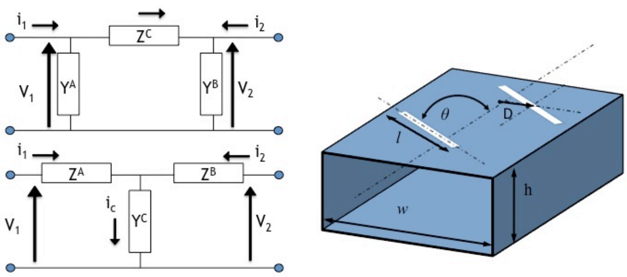

Fig. 2: Π and T networks and compound slot design parameters

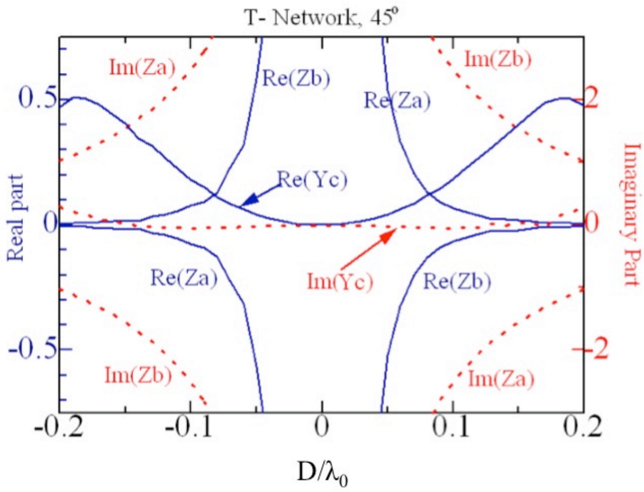

Fig. 3: T-network components for -45º tilted slot, l=0.5λ₀

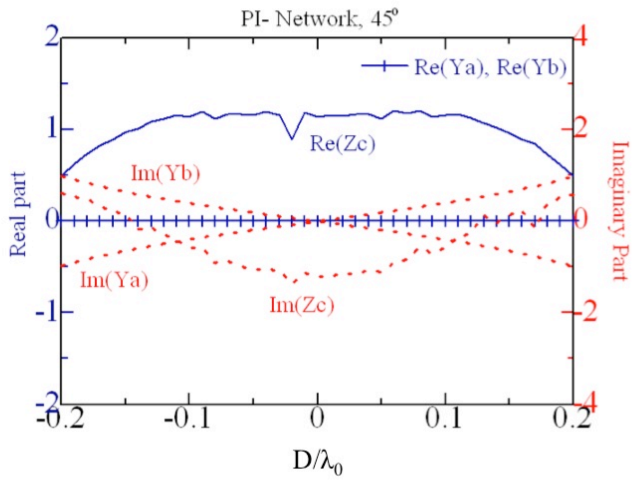

Fig. 4: π-network components for -45º tilted slot, l=0.5λ₀

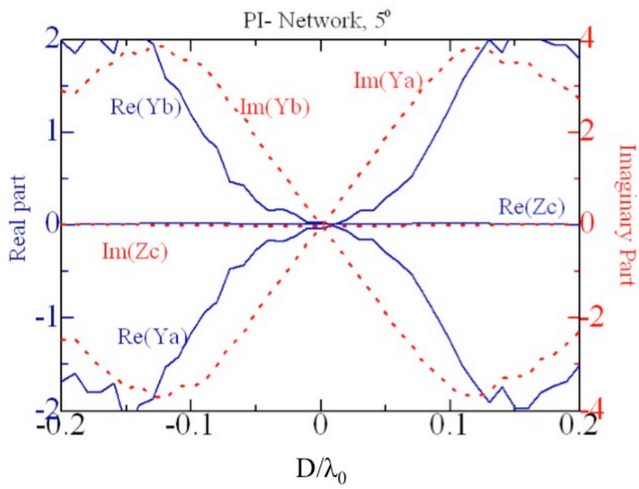

Fig. 5: π-network components for -5º tilted slot, l=0.5λ₀

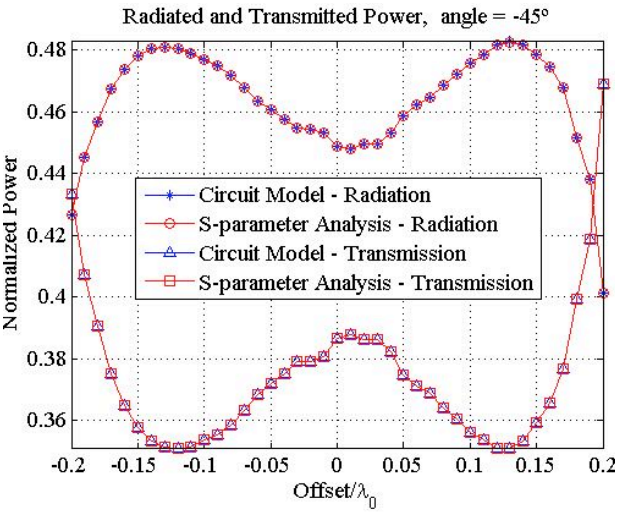

Fig. 6: transmitted and radiated power for -45º compound slot, l=0.5λ_0

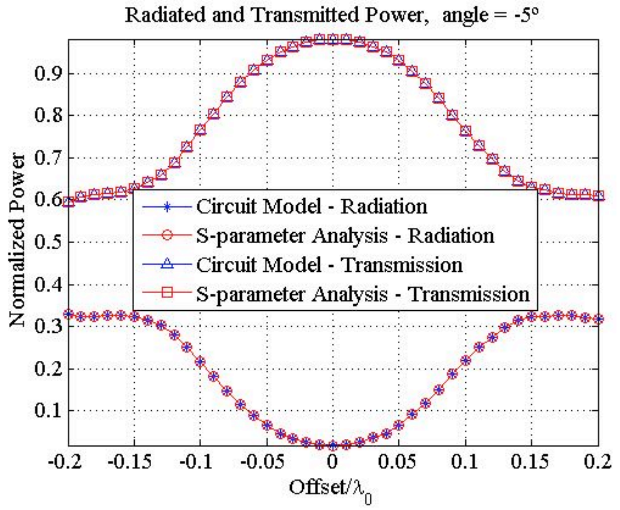

Fig. 7: transmitted and radiated power for -5º compound slot, l=0.5λ_0

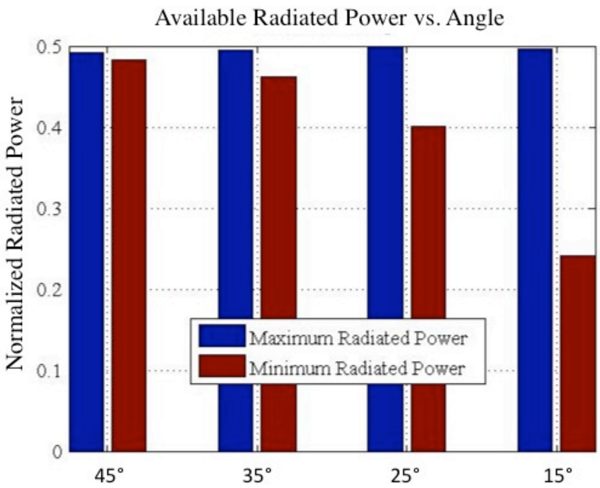

Fig. 8: Maximum and minimum radiation power for 4 different angles at resonance

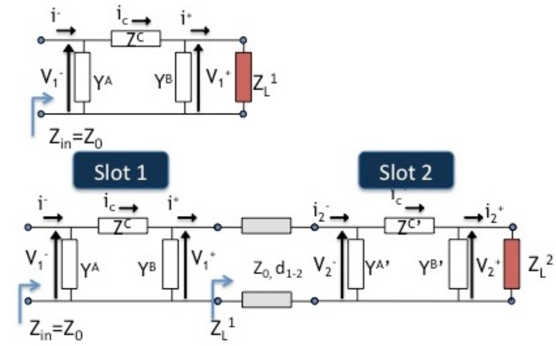

Fig. 9: FMP for the first and second slot.

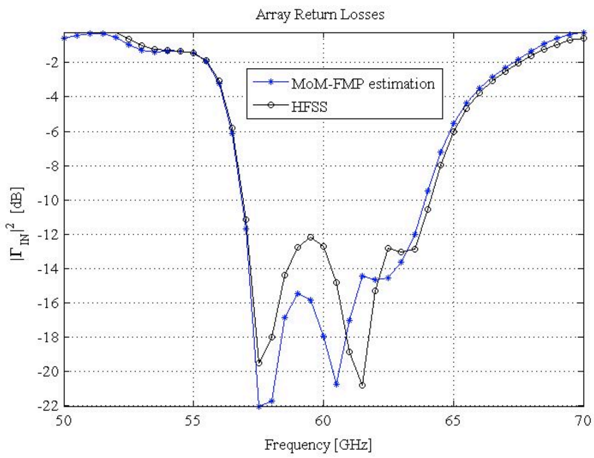

Fig. 10: return losses

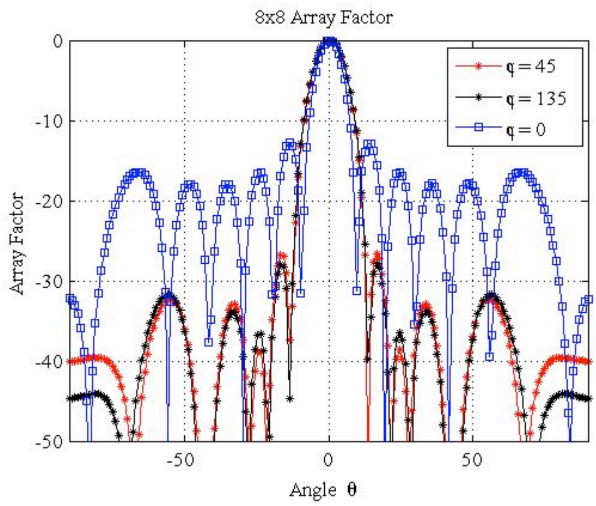

Fig. 11: 8x8 element array factor estimation.